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Energy–momentum tensor in thermal strong-field QED with unstable vacuum

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Abstract
The mean value of the one-loop energy–momentum tensor in thermal QED with an electric-like background that creates particles from vacuum is calculated. The problem is essentially different from calculations of effective actions (similar to the action of Heisenberg–Euler) in backgrounds that respect the stability of vacuum. The role of a constant electric background in the violation of both the stability of vacuum and the thermal character of particle distribution is investigated. Restrictions on the electric field and the duration over which one can neglect the back-reaction of created particles are established.

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(1) Quantum field theory (QFT) in external backgrounds is a good model for the study of quantum processes in cases when a part of a quantized field is strong enough to be treated as a classical one. The validity of that approach is based on the assumption that back-reaction is small. However, from physical reasons, it is clear that back-reaction may be quite strong in external backgrounds that can create particles from vacuum and produce actual work on particles. A complete description of back-reaction is connected to the calculation of the mean energy–momentum tensor (EMT) for the matter field. In this connection, we recall that Heisenberg and Euler computed the mean value of energy density of the Dirac field for a constant weak ($|E| \ll E_c = M^2c^3/|e|\hbar \simeq 1.3 \times 10^{16}$ V cm^{-1}) electromagnetic background at zero temperature and presented the respective one-loop effective Lagrangian $\mathcal{L}$ (see [1]). This Lagrangian was re-introduced by Schwinger [2], without any restrictions on the external field, in order to represent the vacuum-to-vacuum transition amplitude, c_v, as well as the probability, P^v, of a vacuum remaining a vacuum in a constant electric field, namely,

$$c_v = \exp\left(\mathrm{i}\int \mathrm{d}x\,\mathcal{L}\right), \qquad P^v = |c_v|^2 = \exp\{-VT2\,\mathrm{Im}\,\mathcal{L}\}. \tag{1}$$

Here, T is the field duration and V is the observation volume. Nevertheless, the problem of back-reaction can be solved only by computing the mean EMT in the corresponding background. This kind of calculation first appeared in [3] for a spinor field in an arbitrary constant electric background with vacuum initial state, and then was developed and completed in [4], including the case of thermal initial state.

Here, on the basis of these results, we discuss the mean EMT $\langle T_{\mu\nu}(t)\rangle$ of a spinor field in a quasiconstant electric background. Namely, we select the background as a so-called T-constant field, being a uniform constant electric field E, acting only during a sufficiently large but finite time interval T,

$$T \gg T_0, \qquad T_0 = |qE|^{-1/2} + M^2|qE|^{-3/2}, \tag{2}$$

where q is the charge and M is the mass of an electron. The T-constant field turns on at $t_1 = -T/2$; it turns off at $t_2 = T/2$, and is directed along the x^3-axis[3]. In addition, a parallel constant magnetic field B may also be present at the background. We treat the background nonperturbatively and use the techniques developed in [6]–[9].

(2) The mean EMT $\langle T_{\mu\nu}(t)\rangle$ is defined as

$$\langle T_{\mu\nu}(t)\rangle = \mathrm{Tr}[\rho_{\mathrm{in}} T_{\mu\nu}], \tag{3}$$

where ρ_{in} is the density operator of the initial state in the Heisenberg picture; the EMT operator $T_{\mu\nu}$ has the form

$$T_{\mu\nu} = \tfrac{1}{8}\{[\bar\psi(x), \gamma_\mu P_\nu \psi(x)] + [P_\nu^* \bar\psi(x), \gamma_\mu \psi(x)]\} + \tfrac{1}{8}\{\mu \leftrightarrows \nu\},$$

$P_\mu = \mathrm{i}\partial_\mu - qA_\mu(x)$, while $\psi(x)$ denote spinor field operators (in the Heisenberg representation) which obey the Dirac equation with the corresponding external field. The initial state of the system under consideration is selected as an equilibrium state of noninteracting in-particles at the temperature θ with the chemical potentials $\mu^{(\zeta)}$, so that

$$\rho_{\mathrm{in}} = Z^{-1}\exp\left\{\beta\left(\sum_{\zeta=\pm}\mu^{(\zeta)}N^{(\zeta)} - H(t_1)\right)\right\}, \qquad \mathrm{Tr}\,\rho_{\mathrm{in}} = 1, \tag{4}$$

where $\beta = \theta^{-1}$, and $N^{(\zeta)}$ are the in-paricle number operators:

$$N^{(+)} = \sum_n a_n^\dagger(\mathrm{in})a_n(\mathrm{in}), \qquad N^{(-)} = \sum_n b_n^\dagger(\mathrm{in})b_n(\mathrm{in}).$$

It is supposed that in the Heisenberg picture (see [6, 9] for notation) there exists a set of creation and annihilation operators, $a_n^\dagger(\mathrm{in})$, $a_n(\mathrm{in})$ of in-particles (electrons), and similar operators $b_n^\dagger(\mathrm{in})$, $b_n(\mathrm{in})$ of in-antiparticles (positrons), with a corresponding in-vacuum $|0, \mathrm{in}\rangle$, and a set of creation and annihilation operators $a_n^\dagger(\mathrm{out})$, $a_n(\mathrm{out})$, of out-electrons, and similar operators $b_n^\dagger(\mathrm{out})$, $b_n(\mathrm{out})$ of out-positrons, with a corresponding out-vacuum $|0, \mathrm{out}\rangle$. By n we denote a complete set of possible quantum numbers. The in- and out-operators obey the canonical anticommutation relations. The Hamiltonian $H(t)$ of the quantized spinor field is time-dependent due to the external field. It is diagonalized (and also has a canonical form) in terms of the first set at the initial instant of time, and is diagonalized (and has a canonical form) in terms of the second set at the final instant of time.

For example, the quantum Heisenberg field $\psi(x)$ can be expressed in terms of the creation and annihilation operators of in-particles with the help of some appropriate sets of solutions of the Dirac equation with the external field. Namely, the in-particles are

[3] As shown in [5], calculations of particle creation in a T-constant field explain typical features of the effect in a large class of quasiconstant electric backgrounds.

associated with a complete set (in-set) of solutions $\{{}_\zeta\psi_n(x)\}$ with asymptotics ${}_\zeta\psi_n(t_1,\mathbf{x})$ (at the initial time instant t_1) being eigenvectors of the one-particle Dirac Hamiltonian $\mathcal{H}(t)=\gamma^0([M+\boldsymbol{\gamma}(\mathrm{i}\boldsymbol{\nabla}-q\mathbf{A}(t,\mathbf{x}))])$ at $t=t_1$,

$$\mathcal{H}(t_1)\,{}_\zeta\psi_n(t_1,\mathbf{x})=\zeta\varepsilon_n^{(\zeta)}\,{}_\zeta\psi_n(t_1,\mathbf{x}), \tag{5}$$

where $\varepsilon_n^{(\zeta)}$ are the energies of in-particles in a state specified by a complete set of quantum numbers n, and $\varepsilon_n^{(\pm)}>0$. Then,

$$\psi(x)=\sum_n\left[{}_+\psi_n(x)a_n(\mathrm{in})+{}_-\psi_n(x)b_n^\dagger(\mathrm{in})\right] \tag{6}$$

and

$$H(t_1)=\sum_n\left[\varepsilon_n^{(+)}a_n^\dagger(\mathrm{in})a_n(\mathrm{in})+\varepsilon_n^{(-)}b_n^\dagger(\mathrm{in})b_n(\mathrm{in})\right]. \tag{7}$$

Correspondingly, the initial vacuum is defined by $a_n(\mathrm{in})|0,\mathrm{in}\rangle=b_n(\mathrm{in})|0,\mathrm{in}\rangle=0$ for every n.

Using representation (6), we can express the Green function $\mathrm{i}\,\mathrm{Tr}\{\rho_{\mathrm{in}}T\psi(x)\bar\psi(x')\}$ via the in-set of solutions, separating at the same time the temperature-dependent contribution from the vacuum contribution as follows:

$$\mathrm{i}\,\mathrm{Tr}\{\rho_{\mathrm{in}}T\psi(x)\bar\psi(x')\}=S^\theta(x,x')+S^c_{\mathrm{in}}(x,x'), \tag{8}$$

where

$$\begin{aligned}S^c_{\mathrm{in}}(x,x')&=\mathrm{i}\langle 0,\mathrm{in}|T\psi(x)\bar\psi(x')|0,\mathrm{in}\rangle\\&=\theta(x_0-x_0')S^-_{\mathrm{in}}(x,x')-\theta(x_0'-x_0)S^+_{\mathrm{in}}(x,x'),\\S^\mp_{\mathrm{in}}(x,x')&=\mathrm{i}\sum_n{}_\pm\psi_n(x)\,{}_\pm\bar\psi_n(x');\end{aligned} \tag{9}$$

and

$$\begin{aligned}&S^\theta(x,x')=\mathrm{i}\sum_n\left[-{}_+\psi_n(x)\,{}_+\bar\psi_n(x')N_n^{(+)}(\mathrm{in})+{}_-\psi_n(x)\,{}_-\bar\psi_n(x')N_n^{(-)}(\mathrm{in})\right],\\&N_n^{(\zeta)}(\mathrm{in})=\left[\exp\left\{\beta\left(\varepsilon_n^{(\zeta)}-\mu^{(\zeta)}\right)\right\}+1\right]^{-1}.\end{aligned} \tag{10}$$

It is important to stress that due to vacuum instability S^c_{in} differs from the causal Green function

$$S^c(x,x')=i\frac{\langle 0,\mathrm{out}|T\psi(x)\bar\psi(x')|0,\mathrm{in}\rangle}{\langle 0,\mathrm{out}|0,\mathrm{in}\rangle}. \tag{11}$$

This causal Green function and the difference $S^p(x,x')=S^c_{\mathrm{in}}(x,x')-S^c(x,x')$ can be expressed through appropriate solutions of the Dirac equation. To this end, we define that the out-particles be associated with a complete set (out-set) of solutions $\{{}^\zeta\psi_n(x)\}$ with asymptotics ${}^\zeta\psi_n(t_2,\mathbf{x})$ (at the final time instant t_2) being eigenvectors of the one-particle Dirac Hamiltonian $\mathcal{H}(t_2)$. The out-set can be decomposed in the in-set as follows:

$${}^\zeta\psi(x)={}_+\psi(x)G({}_+|{}^\zeta)+{}_-\psi(x)G({}_-|{}^\zeta), \tag{12}$$

where the coefficients $G({}_\zeta|{}^{\zeta'})$ are expressed via inner products of these sets. These coefficients obey unitary conditions that follow from normalization conditions imposed on the solutions. From (11), it follows that

$$\begin{aligned}S^c(x,x')&=\theta(x_0-x_0')S^-(x,x')-\theta(x_0'-x_0)S^+(x,x'),\\S^-(x,x')&=\mathrm{i}\sum_{n,m}{}^+\psi_n(x)G({}_+|{}^+)^{-1}_{nm}\,{}_+\bar\psi_m(x'),\\S^+(x,x')&=\mathrm{i}\sum_{n,m}{}_-\psi_n(x)[G({}_-|{}^-)^{-1}]^\dagger_{nm}\,{}^-\bar\psi_m(x'),\end{aligned} \tag{13}$$

see [10, 6]. Then, the difference $S^p(x,x') = S^c_{\rm in}(x,x') - S^c(x,x')$ has the form

$$S^p(x,x') = \mathrm{i}\sum_{nm} {}_-\psi_n(x)[G({}_+|{}^-)G({}_-|{}^-)^{-1}]^\dagger_{nm}\,{}_+\bar\psi_m(x'). \tag{14}$$

This function vanishes in the case of a stable vacuum, since it contains the coefficients $G({}_+|{}^-)$ related to the mean number of created particles.

Thus, the mean EMT (3) can be presented in such a form that contributions of different kinds are explicitly separated, namely,

$$\langle T_{\mu\nu}(t)\rangle = \langle T_{\mu\nu}(t)\rangle^0 + \langle T_{\mu\nu}(t)\rangle^\theta, \qquad \langle T_{\mu\nu}(t)\rangle^0 = \mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c + \mathrm{Re}\langle T_{\mu\nu}(t)\rangle^p. \tag{15}$$

The quantities $\langle T_{\mu\nu}(t)\rangle^{c,p,\theta}$ are defined as

$$\begin{aligned} &\langle T_{\mu\nu}(t)\rangle^{c,p,\theta} = \mathrm{i}\,\mathrm{tr}[A_{\mu\nu}S^{c,p,\theta}(x,x')]|_{x=x'}, \\ &A_{\mu\nu} = 1/4[\gamma_\mu(P_\nu + P'^*_\nu) + \gamma_\nu(P_\mu + P'^*_\mu)], \end{aligned} \tag{16}$$

where $\mathrm{tr}[\cdots]$ is the trace in the space of 4×4 matrices.

Calculating components $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c$ in the case of the T-constant field being subject to condition (2), we find that they are connected with the real part of the Heisenberg–Euler Lagrangian $\mathcal{L}$, namely,

$$\begin{aligned} &\mathrm{Re}\langle T_{00}(t)\rangle^c = -\mathrm{Re}\langle T_{33}(t)\rangle^c = E\frac{\partial\,\mathrm{Re}\,\mathcal{L}}{\partial E} - \mathrm{Re}\,\mathcal{L}, \\ &\mathrm{Re}\langle T_{11}(t)\rangle^c = \mathrm{Re}\langle T_{22}(t)\rangle^c = \mathrm{Re}\,\mathcal{L} - B\frac{\partial\,\mathrm{Re}\,\mathcal{L}}{\partial B}. \end{aligned} \tag{17}$$

These components describe the contribution due to the vacuum polarization. Their field-dependent parts are finite after a standard renormalization and do exist for an arbitrary quasiconstant electric field. They are local, i.e., they depend on t, but do not depend on the history of the process. The contributions $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^p$ arise due to vacuum instability. The quantity $\langle T_{\mu\nu}(t)\rangle^\theta$ presents the contribution due to the existence of the initial thermal distribution.

(3) We have calculated the functions S^p and S^θ explicitly (see the details in [4]). Using these functions, we have obtained a representation for the mean EMT $\langle T_{\mu\nu}(t)\rangle$. The expression consists of three terms:

$$\langle T_{\mu\nu}(t)\rangle = \mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c + \mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c_\theta + \tau^p_{\mu\nu}(t). \tag{18}$$

As mentioned above, the term $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c$ is the contribution due to vacuum polarization. The term $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c_\theta$ describes the contribution due to the work of the external field on the particles at the initial state. The term $\tau^p_{\mu\nu}(t)$ describes the contribution due to particle creation from vacuum. The two latter contributions depend on the time interval $t-t_1$, and therefore they are global quantities. The term $\tau^p_{\mu\nu}(t)$ includes the factor $\exp\{-\pi M^2/|qE|\}$. This factor is exponentially small for a weak electric field, $M^2/|qE| \gg 1$, whereas the term is not small as long as the electric field strength approaches the critical value E_c. On the other hand, the term $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c_\theta$, as well as $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c$, does exist in an arbitrary electric field. When the T-constant electric field turns off (at $t > t_2$), the local contribution to $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c$ made by the electric field becomes equal to zero; whereas the global contributions, given by $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c_\theta$ and $\tau^p_{\mu\nu}(t)$, do not vanish and retain their values at any $t > t_2$.

In the case of a strong electric field, we separate the increasing terms related to a large interval $t-t_1$ and call them the 'leading contributions'. They appear due to particle creation. However, the quantities $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle^c_\theta$ are computed for any t, $0 \leqslant t-t_1 \leqslant T$. All these contributions are investigated in detail at different regimes and limits of weak and strong fields, as well as at low and high temperatures. For all these limiting cases, we have obtained the leading contributions, which are given by elementary functions of the fundamental dimensionless parameters. These results are presented below.

In a weak electric field, $|qE|/M^2 \ll 1$, at low temperatures, $M\beta \gg 1$, for $|\mu^{(\zeta)}| \ll M$, the nonzero expressions for $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle_\theta^c$ read as follows:

(a) when the increment of kinetic momentum is small, $|qE|(t-t_1)/M \ll 1$, we have

$$\mathrm{Re}\langle T_{\mu\nu}(t)\rangle_\theta^c = \bar{T}^0_{\mu\nu} + \Delta T_{\mu\nu}, \tag{19}$$

and

$$\begin{aligned}
\Delta T_{11} &= \Delta T_{22} = -C\frac{2}{M\beta}\left[1+\frac{M(t-t_1)^2}{\beta}\right]\left[1+O\left(\frac{1}{M\beta}\right)\right],\\
\Delta T_{33} &= C\left[-\frac{3}{2M\beta}+4\frac{M(t-t_1)^2}{\beta}\right]\left[1+O\left(\frac{1}{M\beta}\right)\right],\\
\Delta T_{00} &= C\left[-\frac{235}{256}+2\frac{M(t-t_1)^2}{\beta}\right]\left[1+O\left(\frac{1}{M\beta}\right)\right],\\
C &= \frac{(qE)^2}{2\pi^2}\left(\frac{\pi}{2M\beta}\right)^{1/2}\mathrm{e}^{-M\beta},
\end{aligned} \tag{20}$$

where we have explicitly separated the part $\bar{T}^0_{\mu\nu} = \langle T_{\mu\nu}(t)\rangle_\theta^c|_{E=0}$, being independent of the electric field;

(b) when the increment of kinetic momentum is large, $|qE|(t-t_1)/M \gg 1$, we have

$$\begin{aligned}
\mathrm{Re}\langle T_{11}(t)\rangle_\theta^c &= \mathrm{Re}\langle T_{22}(t)\rangle_\theta^c = \frac{M}{|qE|(t-t_1)}\bar{T}^0_{11},\\
\mathrm{Re}\langle T_{00}(t)\rangle_\theta^c &= \mathrm{Re}\langle T_{33}(t)\rangle_\theta^c = |qE|(t-t_1)\sum_{\zeta=\pm} n^{(\zeta)},
\end{aligned} \tag{21}$$

where $n^{(\zeta)}$ is the initial particle density, and $\bar{T}^0_{11}$ is the initial value of $\mathrm{Re}\langle T_{11}(t)\rangle_\theta^c$ at $t < t_1$.

At high temperatures, $M\beta \ll 1$, $\sqrt{|qB|}\beta \ll 1$, $\sqrt{|qE|}\beta \ll 1$, and in case the increment of kinetic momentum is small, $|qE|(t-t_1)\beta \ll 1$, the nonzero expressions for $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle_\theta^c$ have the form (19), while the terms $\Delta T_{\mu\nu}$ depending on the electric field have the form

$$\begin{aligned}
\Delta T_{11} &= \Delta T_{22} = \frac{(qE)^2}{12\pi^2}\left[-\frac{1}{2}\ln(M\beta)+O(1)-\frac{29\pi^2}{15}\frac{(t-t_1)^2}{\beta^2}\right],\\
\Delta T_{33} &= \frac{(qE)^2}{12\pi^2}\left[-\ln(M\beta)+O(1)-\frac{7\pi^2}{15}\frac{(t-t_1)^2}{\beta^2}\right],\\
\Delta T_{00} &= \frac{(qE)^2}{12\pi^2}\left[-2\ln(M\beta)+O(1)-\frac{13\pi^2}{3}\frac{(t-t_1)^2}{\beta^2}\right].
\end{aligned} \tag{22}$$

In case the increment of kinetic momentum is large, $|qE|(t-t_1)\beta \gg 1$, the nonzero expressions for $\mathrm{Re}\langle T_{\mu\nu}(t)\rangle_\theta^c$ decrease exponentially:

$$\mathrm{Re}\langle T_{\mu\mu}(t)\rangle_\theta^c \sim \exp[-|qE|(t-t_1)\beta]. \tag{23}$$

Before the T-constant field turns on, the system under consideration is at thermal equilibrium. The system is described by the thermodynamic potential $\Omega = -\theta\ln Z$, as well as by the temperature- and field-dependent renormalized effective Lagrangian $\mathcal{L}^\theta = -\Omega/V$, where the partition function Z is defined by (4). The quantity $\Omega|_{B=0}$ is well known from textbooks. In case $\mu^{(+)} = -\mu^{(-)} = \mu$, the expression for $\mathcal{L}^\theta$ was presented in [11, 12], and at $\mu = 0$ was obtained in [13]. It turns out that the nonzero components of $\bar{T}^0_{\mu\nu}$ in (19) can be deduced from the effective Lagrangian $\mathcal{L}^\theta$ as follows:

$$\begin{aligned} \bar{T}^0_{00} &= -\mathcal{L}^\theta - \beta\frac{\partial\mathcal{L}^\theta}{\partial\beta} + \sum_{\zeta=\pm}\mu^{(\zeta)}\frac{\partial\mathcal{L}^\theta}{\partial\mu^{(\zeta)}}, \\ \bar{T}^0_{11} &= \bar{T}^0_{22} = \mathcal{L}^\theta - B\frac{\partial\mathcal{L}^\theta}{\partial B}, \qquad \bar{T}^0_{33} = \mathcal{L}^\theta. \end{aligned} \tag{24}$$

Thus, our results for $\langle T_{\mu\nu}(t)\rangle^c_\theta$ are in agreement with the results obtained previously in the case $E = 0$.

For $t \geqslant t_2$, particle production is absent, and the quantity $\tau^p_{\mu\nu}(t_2)$ presents the mean EMT of created pairs. At low temperatures, $\beta\left|\left(\varepsilon^{(\zeta)}_{\mathbf{p}} - \mu^{(\zeta)}\right)\right| \gg 1$, under the assumption $\varepsilon^{(\zeta)}_{\mathbf{p}} > |\mu^{(\zeta)}|$, the leading contributions in $\tau^p_{\mu\nu}(t_2)$ coincide with the vacuum contributions:

$$\begin{aligned} &\tau^p_{\mu\nu}(t_2) = \tau^{cr}_{\mu\nu}(t_2), \qquad \tau^{cr}_{00}(t_2) = \tau^{cr}_{33}(t_2) = |qE|Tn^{cr}, \\ &\tau^{cr}_{11}(t_2) = \tau^{cr}_{22}(t_2) \sim \ln\left(\sqrt{|qE|}T\right). \end{aligned} \tag{25}$$

Here, n^{cr} is the total number-density of pairs created by the T-constant electric field,

$$n^{cr} = \frac{q^2}{4\pi^2}EBT\coth(\pi B/E)\exp\left(-\pi\frac{M^2}{|qE|}\right).$$

At high temperatures, $\beta|qE|T \ll 1$, we have

$$\begin{aligned} &\tau^p_{00}(t_2) = \tau^p_{33}(t_2) = \tfrac{1}{6}\beta|qE|T\tau^{cr}_{00}(t_2), \\ &\tau^p_{11}(t_2) = \tau^p_{22}(t_2) = \tfrac{1}{2}\beta|qE|T\tau^{cr}_{11}(t_2). \end{aligned} \tag{26}$$

(4) The action of electric field manifests itself differently for different components of the EMT $\langle T_{\mu\nu}(t)\rangle$. One can see that in a strong electric field, or in a field acting during a long time interval, the leading terms for the energy density, $\mathrm{Re}\langle T_{00}(t)\rangle^c_\theta + \tau^p_{00}(t)$, and those for the pressure along the direction of the electric field, $\mathrm{Re}\langle T_{33}(t)\rangle^c_\theta + \tau^p_{33}(t)$, are identical. However, for vacuum polarization, these terms have opposite signs: $\mathrm{Re}\langle T_{00}(t)\rangle^c = -\mathrm{Re}\langle T_{33}(t)\rangle^c$. This is due to a difference between the equation of state for the relativistic fermions and the equation of state for the electromagnetic field. The dependence of the electric field and its duration for the transversal component $\mathrm{Re}\langle T_{11}(t)\rangle^c_\theta + \tau^p_{11}(t)$ is quite different from the dependence of the longitudinal component $\mathrm{Re}\langle T_{33}(t)\rangle^c_\theta + \tau^p_{33}(t)$. This implies that one cannot define a universal functional of the electric field whose variations should produce all the components of EMT. This fact holds equally true for vacuum and non-vacuum initial states. We believe that for particle-creating backgrounds one cannot obtain any generalization of the Heisenberg–Euler Lagrangian for the mean values. This fact is connected with the existence of non-local contributions to the EMT. This may explain the failure of numerous attempts at considering one-loop effects on the basis of different variants of such a generalization. Our results also demonstrate that in particle-creating backgrounds any generalization of the Heisenberg–Euler Lagrangian to the case of finite temperatures is problematic.

One can neglect the back-reaction of created pairs in a strong electric field only in the case $\tau^p_{00}(t_2) \ll E^2/8\pi$, which implies a restriction on the strength and the duration of the electric field. This restriction has the same form $|qE|T^2 \ll \frac{\pi^2}{2q^2}$ for both the initial vacuum state and the low-temperature initial thermal state. On the other hand, there exists another restriction, $1 \ll |qE|T^2$ (see (2)), which allows one to disregard the details of switching the electric field on and off. Collecting these two restrictions, we obtain a range of the dimensionless parameter $|qE|T^2$ for which QED with a strong constant electric field is consistent:

$$1 \ll |qE|T^2 \ll \pi^2/2q^2.$$

Similar restrictions can take place when the initial thermal equilibrium is examined at sufficiently high temperatures, $\beta|qE|T \ll 1$. In this case, we have two inequalities, $\beta|qE|^2T^3 \ll \frac{3\pi^2}{q^2}$ and $1 \ll |qE|T^2$, which imply

$$1 \ll |qE|T^2 \ll \frac{3\pi^2}{q^2\beta|qE|T}.$$

We can see that the upper restriction for $|qE|T^2$ is weaker than it is in the low-temperature case.

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